Practices 1, 2

Theme: **COMPOSITION of the MATHEMATICAL DESCRIPTION of ELEMENTS ACS**

According to accepted in the theory of regulation and the controls of ways composite the mathematical description (model) ACS as differential, algebraic etc. equations.

In the given section show the composition of the mathematical description (model) of the correction device which is looking like *R-C-L* - chains is considered. The basic circuit of the correction device and the construction data are given.

The integrated algorithm of composition of the mathematical description (model) of the correction device:

1) To allocate contours *R-C-L* - chains.

2) To write down for each contour of the equation of balance using the second law of Kirkhgoff. It is necessary to remember, that the falling powers on active, inductive and capacitance are equal $i(t)R$, $L\frac{di}{dt}$, $\frac{1}{c}\int i(t)dt$ accordingly (R, L, C) the given constant construction data of the device.

3) To write down system of the equations for contours in the operated form, using direct integrated transformation of Laplace:

$$i(t) \to I(s); \qquad \frac{di}{dt} \to s\,I(s) \;\; ; \int i(t)dt \to \frac{1}{s}I(s).$$

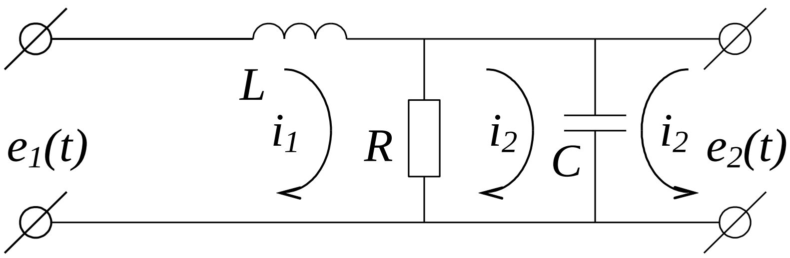
4) To copy system of the equations, having entered designations for factors by *I(s)* currents, through *Z (s)*. For example, $Z(s) = (R + Ls + 1/(cs))$.

5) To composite transfer function as:

$$W(s) = \frac{E_2(s)}{E_1(s)}.$$

6) Than it is possible to write down using return transformation of Laplace L^{-1} $\{E_2$ (s)$\}$ = L^{-1} $\{E_1$(s)$\}$, dependence of output coordinate $e_2(t)$ on input $e_1(t)$ and construction data of the device.

Example. The basic circuit of the correction device looks like



Consider construction data of the device constant and given.

Composite the mathematical description (model) of the device; to take e_1 (t) for an input and take e_2 (t) for an output.

Algorithm and solution

1) Write down for each contour of the equation of balance using the second law of Kirkhgoff.

$$\begin{cases} L\dfrac{di_1}{dt} + i_1(t)R - i_2(t)R = e_1(t) \\ i_2(t)R + \dfrac{1}{c}\displaystyle\int i_2(t)dt - i_1(t)R = 0 \\ \dfrac{1}{c}\displaystyle\int i_2(t)dt = e_2(t) \end{cases}$$

2) Write down system of the equations for contours in the operated form, using direct integrated transformation of Laplace:

$$\begin{cases} E_1(s) = I_1(s)Ls + I_1(s)R - I_2(s)R \\ 0 = I_2(s)R + \dfrac{1}{cs}I_2(s) - I_1(s)R \\ E_2(s) = \dfrac{1}{cs}I_2(s) \end{cases}$$

3) We make elementary conversions and we make designations.

$$\begin{cases} E_1(s) = I_1(s)(Ls + R) - I_2(s)R \\ 0 = I_2(s)(R + \dfrac{1}{cs}) - I_1(s)R \\ E_2(s) = \dfrac{1}{cs}I_2(s) \end{cases}$$

$$\begin{cases} E_1(s) = I_1(s)z_1(s) - I_2(s)z_2(s) \\ 0 = I_2(s)z_3(s) - I_1(s)z_2(s) \\ E_2(s) = I_2(s)z_4(s) \end{cases}$$

where $z_1(s) = Ls+R$; $z_2(s) = R$; $z_3(s) = R+\dfrac{1}{cs}$; $z_4(s) = \dfrac{1}{cs}$.

4) We will express currents of $I_1(s)$ *through* $I_2(s)$ from the second expression and we will add in the first and second expressions; we will receive transfer function of the correcting device.

$I_1(s) = I_2(s)\frac{z_3(s)}{z_2(s)}$

$$\begin{cases} E_1(s) = I_2(s)\left(\frac{z_1(s)z_3(s)}{z_2(s)} - z_2(s)\right) \\ E_2(s) = I_2(s)z_4(s) \end{cases}$$

$$W(s) = \frac{E_2(s)}{E_1(s)} = \frac{I_2(s)z_4(s)z_2(s)}{I_2(s)(z_1(s)z_3(s) - z_2^2(s)}$$

5) We will receive the equation of dependence of output coordinate of the correcting device (mathematical model) in the operator form.

$E_2(s) = W(s)\ E_1(s)$

$$E_2(s) = \frac{z_4(s)z_2(s)}{z_1(s)z_3(s)z_2^2(s)} E_1(s)$$

$$E_2(s) = \frac{\frac{1}{cs}R}{(Ls+R)(R+\frac{1}{cs}) - R^2} E_1(s)$$

$$[(Ls+R)\ (R+\frac{1}{cs}) - R^2]E_2(s) = \frac{1}{cs}R\ E_1(s)$$

$$[LsR + \frac{1}{c}L + \frac{1}{cs}R]\ E_2(s) = \frac{1}{cs}R\ E_1(s)$$

6) We will receive the equation of dependence of output coordinate of the correcting device (mathematical model) in a time domain, using inverse transformation of Laplace.

$$L^{-1}\{(Ls^2R+\frac{1}{c}L\ s+\frac{1}{c}R\ )\ E_2(s)\} = L^{-1}\{\frac{R}{c}\ E_1(s)\}$$

$$LR\frac{d^2e_2}{dt^2} + \frac{L}{c}\ \frac{de_2}{dt} + \frac{R}{c}e_2(t) = \frac{R}{c}\ e_1(t)$$

$$Lc\frac{d^2e_2}{dt^2} + \frac{L}{R}\ \frac{de_2}{dt^2} + e_2(t) = e_1(t)$$

$$a_0\frac{d^2e_2}{dt^2} + a_1\frac{de_2}{dt} + e_2(t) = e_1(t),$$

where $a_0 = Lc$; $a_1 = \frac{L}{R}$.

Answer: the mathematical description (mathematical model) of the correcting device is received

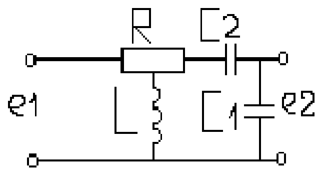
$$a_0\frac{d^2e_2}{dt^2} + a_1\frac{de_2}{dt} + e_2(t) = e_1(t),$$

где $a_0 = Lc$; $a_1 = \frac{L}{R}$.

Task (on variant):

The basic circuit of the correcting device with constant design data is set. Receive the mathematical description (model) of the correcting device; take *$e_1(t)$* for an input, to take *$e_2(t)$* for an output.

Variants *(numbering of circuits from top to down and from left to right):*



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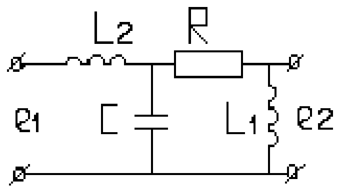

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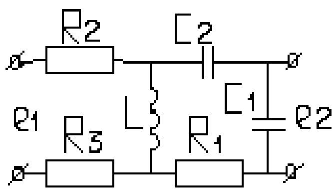

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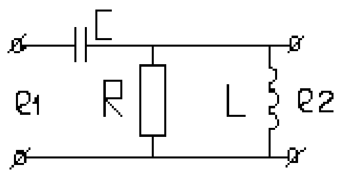

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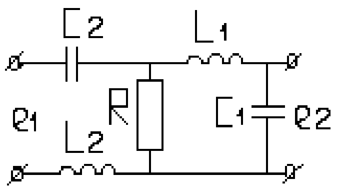

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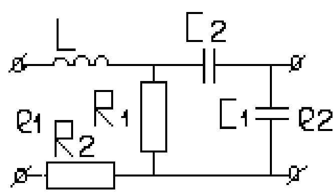

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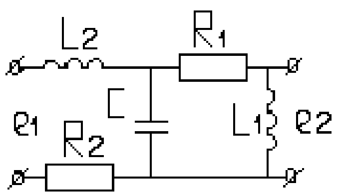

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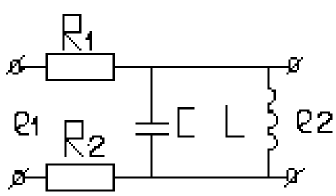

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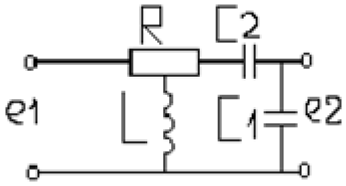

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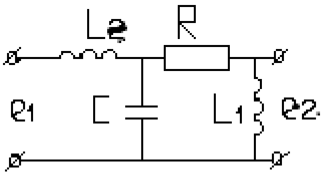

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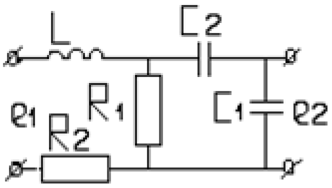

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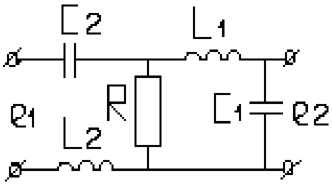

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